

Signalized Intersection Analysis and Design – Implications of Day-to-Day Variability in Peak Hour Volumes on Delay

Bruce Hellinga¹, PhD, PEng and Zeeshan Abdy²

1. Associate Professor
(Corresponding Author)
Department of Civil and Environmental Engineering,
University of Waterloo
Waterloo, Ontario, N2L 3G1, Canada
Tel: (519) 888-4567 ext. 2630
Fax: (519) 888-4349
E-mail: bhellinga@uwaterloo.ca

2. PhD Candidate
Department of Civil and Environmental Engineering,
University of Waterloo
Waterloo, Ontario, N2L 3G1, Canada
Tel: (519) 888-4567 ext. 6596
Fax: (519) 888-4349
E-mail: zraby@uwaterloo.ca

(Accepted for publication in the ASCE Journal of Transportation)

ABSTRACT

Traffic signal timing plans are typically developed on the basis of turning movement traffic and pedestrian volume counts aggregated to 15-minute intervals and obtained over a 4 or 8 hour period on a single day. These data are used to identify the peak hour and to compute the peak hour turning movement traffic volumes. They may also be used to compute the peak hour factor (*PHF*). These values are then used as input to the analysis methods defined in the Highway Capacity Manual or within popular signal timing optimization software to estimate signal performance in terms of expected average vehicle delay. Delay estimation methods explicitly consider several sources of variability (e.g. an assumed distribution of individual vehicle headways in the arrival traffic stream). However, these methods do not consider the day-to-day variability that exists within key delay estimation parameters such as the *PHF* and peak hour traffic volume. The lack of consideration of this variability may be because (a) it is assumed that the impact of the variability is small; and/or (b) methods have not been developed by which the variability can be considered.

This paper presents findings of a study that quantifies the impact of day-to-day variability of intersection peak hour approach volumes on intersection delay and demonstrates that this impact is not insignificant and therefore should not be ignored. Finally, the study explores the number of days for which intersection approach volumes should be counted in order to establish intersection delay within a desired level of confidence. The results indicate that for intersections operating near capacity 3 days of peak hour volume observations are required to estimate the average intersection delay with an estimation error of 50% of the true mean and 7 days of traffic counts are required to estimate intersection delay with an error of 30% of the true mean.

CEDB SUBJECT HEADINGS: Signals, Traffic Delay, flow, Volume Change

INTRODUCTION

In North America, the Highway Capacity Manual (HCM 2000) is the most widely adopted method for analysis of signalized intersections. The HCM defines signalized intersection performance in terms of average vehicle delay (seconds per vehicle) and then maps this delay against predefined boundaries to define intersection performance in terms of six levels of service (i.e. LOS A through LOS F).

Intersection performance as measured by delay is a function of many factors including, signal timing plan, turning movement traffic demands, traffic stream composition, pedestrian volumes, intersection geometry, temporal variation in traffic demands, the headway distribution of each traffic stream, driver characteristics, weather and road surface conditions and visibility. Some of these factors are invariant for a given intersection operating under a defined signal control strategy (e.g. geometry and signal timing plan) while others vary (e.g. weather, traffic demands, etc.).

Some of this variability is captured (or controlled for) by the intersection analysis methodology. For example, traffic demands vary by time of day, but this is controlled for by applying the analysis method for the peak hour volume and utilizing the peak hour factor (PHF). Weather conditions are controlled for by assuming ideal weather conditions. The random variability of vehicle arrivals (i.e. headway distribution of the approach traffic streams) is considered by assuming Poisson arrivals and then the influence of nearby upstream signalized intersections in terms of creating platoons is considered.

However, variability of other factors is not considered including the day-to-day variability in the peak hour traffic volumes, PHF, and saturation flow rate. This raises a number of issues about appropriate criteria for intersection control evaluation and design including:

1. Traffic engineers typically design signal timing plans to achieve a prescribed level of service (say LOS C) during some defined time period (say the peak hour). Generally, this is interpreted to mean that the timing is designed to provide, *on average* over many weekdays, LOS C during the peak hour. But what is the distribution of the performance provided by the signalized intersection? For example, consider two intersections (*A* and *B*) with identical geometry and signal timing plans. Each intersection experiences a mean (calculated as the intersection delay during the peak hour averaged over many weekdays) delay of 20 second/vehicle. Using conventional intersection evaluation methods, one would conclude that these two intersections have equal performance. However, consider the day-to-day variation in performance. Intersection *A* experiences a delay of 5 seconds/vehicle 50% of the time and a delay of 35 seconds/vehicle the remaining 50% of the time. Intersection *B* experiences a delay of 18 seconds/vehicle 50% of the time and a delay of 22 seconds/vehicle the remaining 50% of the time. Given this information, do these two intersections provide the same performance?
2. Signal designs are determined on the basis of turning movement volumes that are assumed to reflect average peak hour demands. Consequently, the intersection delay that is estimated from the average peak hour demands is assumed to reflect the intersection delay during the peak hour if averaged over many days. Is this assumption correct? If it is not correct, what is the level of error introduced by this assumption?
3. In practice, traffic engineers typically collect turning movement volume count data in 15 minute intervals over a peak period (e.g. 2 hour). On the basis of these data, the peak hour is identified, the peak hour volumes are extracted, and the peak hour factor (*PHF*) is computed (or a default value of *PHF* calibrated for the local community if used). If the peak hour of each weekday is considered as a single outcome (or observation) then signal analysis and design are generally conducted on the basis of turning movement counts that represent a

single observation from a distribution. If a certain level of accuracy is desired in terms of estimating the average intersection delay, then over how many days should turning movement counts be obtained?

This paper seeks to address the following specific questions that begin to address these issues:

1. What degree of day to day variability exists in the peak hour traffic volume and to what extent are traffic volumes on different intersection approaches statistically correlated?
2. What impact does the day to day variation in the peak hour volume have on intersection performance as measured by delay?
3. For how many days are turning movement counts required in order to estimate intersection performance with a given level of confidence?

In this paper we answer these questions using empirical data to quantify the distribution of day-to-day peak hour traffic volumes and the degree of statistical correlation between approach volumes. Then these data are used as input to a Monte Carlo simulation to determine the associated distribution of intersection delay.

The next section provides the background of the HCM delay estimation expressions, previous work examining the sensitivity of intersection performance to variability of key input parameters, and the study methodology.

Later sections provide a description of the data used in the study, the characterization of the empirical distributions, the results of the Monte Carlo simulation and finally provide conclusions and recommendations.

ANALYSIS METHODOLOGY

BACKGROUND

Signalized intersections typically form the capacity bottlenecks in urban road networks. Signal timing plans are developed in order to segregate potentially conflicting movements at a signalized intersection. Methods to analyze the performance of a given signal timing plan, and to develop optimal plans, have been developed since the 1950s and are now embedded in design manuals such as the Highway Capacity Manual (HCM) and Canadian Capacity guide (Teply et al., 1995). Most of these methods are based on the pioneering work by Webster (1958) who developed an expression for average delay that captured delay from two sources, namely: (1) delay assuming deterministic arrival and deterministic service rate; and (2) delay assuming random (i.e. Poisson) arrivals and deterministic service rate. Webster's original formulation was modified to permit application to oversaturated conditions and more recent modifications (e.g. HCM 2000) include a term to account for initial queues. In all of these delay expressions, the only source of randomness is in the distribution of headways in the arrival stream. The mean arrival rate is assumed to be constant over the analysis period.

The delay expression, incorporated within the HCM (TRB, 2000), and used within this study to estimate delay, is given by

$$d = d_1(PF) + d_2 + d_3 \quad (1)$$

$$PF = ((1-P)f_{PA}) / (1-(g/C)) \quad (2)$$

$$d_1 = (0.5C (1-(g/C))^2) / (1-\min(1, X)(g/C)) \quad (3)$$

$$d_2 = 900T ((X-1) + \sqrt{(X-1)^2 + ((8kLX)/(cT))}) \quad (4)$$

$$d_3 = (1800Q_b(1+u)t)/(cT) \quad (5)$$

where:

- c = lane group capacity (vehicle per hour (vph)),
- C = cycle length (seconds),
- d = control delay per vehicle (seconds/veh),
- d_1 = uniform control delay assuming uniform arrivals (seconds/veh),
- d_2 = incremental delay to account for randomness (seconds/veh),
- d_3 = initial queue delay (seconds/veh),
- f_{PA} = supplemental adjustment factor for platoon arriving during green,
- g = duration of green interval (seconds),
- g/C = proportion of green time available,
- k = incremental delay based on controller settings,
- l = upstream filtering / metering adjustment factor, and
- PF = progression adjustment factor
- P = proportion of vehicles arriving on green
- T = analysis period (hour),
- X = lane group v/c ratio or degree of saturation,
- Q_b = initial queue at the start of period T (veh),
- u = delay parameter,
- t = duration of unmet demand in T (h).

As is evident from Equations 2 through 5, delay in the HCM is primarily a function of volume and capacity. Volume is typically the hourly flow rate associated with the peak 15-minutes (i.e. volume = peak hour volume/ PHF). Capacity is a function of the signal timing (i.e. g/C ratio) and the saturation flow rate.

However, no consideration is given to the distributions of these inputs and only single point estimates are used.

PREVIOUS RESEARCH

The problems of estimating delays at signalized intersections have been extensively studied in the literature; however the vast majority of the work has focused on developing models for estimating the mean delay - a point estimate of stochastic delays. Detailed discussions of these average delay prediction models have been provided by Allsop (1972), Newell (1982) and Hurdle (1984).

Some work has been done to investigate the variability of delay at signalized intersections. Several studies have developed analytical expressions for the variance of delay. Cronje (1983) and Olszewski (1993, 1994) developed a Markov-chain model to calculate the average delay and time-dependant distribution of average cyclic delay. Fu and Hellings (2000) developed an analytical model of the variance of control delay on the basis of simulated data. Engelbrecht et al. (1997) developed a generalized model for mean control delay and also investigated the variability of delay using simulation. In all of these studies the variability in delay is solely a result of the variability in the time headways of vehicles arriving at the stop line. The mean arrival rate, saturation flow rate, and signal timings are all assumed to be deterministic and constant.

Several studies have also examined the variability of delay on the basis of field data. Teply and Evans (1989) analyzed the delay distribution at a signalized approach for evaluating signal progression quality. They observed that most of the delay distributions are bimodal and a point estimator is not adequate to describe these distributions. Details pertaining to the field data collection effort (i.e. the number of days and time of day over which field data were collected) are not provided in the paper so it is difficult to ascertain the cause for the observed variability. However, given that

the study was conducted to evaluate signal progression, it seems likely that the majority of the observed variability in delay was a result of the time of vehicle arrivals.

More recently, Colyar and Roupail (2003) examined the variability in control delay on a signalized arterial corridor. They observed that when the mean control delay was relatively small - in the Level of Service (LOS) A-B range, the distribution of control delay had a single peak; however, for larger mean delays, the distribution was increasingly bi-modal. Data were collected during the AM peak (7-9 AM) and PM peak (4-6 PM) periods over a number of different days. The authors consider the possibility that traffic volumes vary by time of day (though they do not consider the possibility that traffic volumes vary from one day to the next) but conclude that these changes in volume (within the two hour peak period) are small and therefore the observed variability in control delay is predominantly due to the stochastic nature of vehicle arrivals on a cycle-by-cycle basis.

A recent study conducted by Sullivan et al., (2006) specifically examined the impact of day to day variations in peak hour traffic volumes on intersection service levels. Using weekday data from 22 directional continuous traffic counting stations in the city of Milwaukee, the authors computed the coefficient of variation (COV), computed as standard deviation divided by the mean, of peak hour traffic volume. They found that the COV ranged from 0.048 to 0.155 with a mean of 0.089.

Using this COV in peak hour volumes, they examined the impact on a hypothetical intersection approach controlled by a fixed time signal with a 90 second cycle length and an assumed saturation flow rate of 1900 vph. The approach delay was estimated using the HCM method for mean peak hour volumes, the 85th percentile volume (i.e. mean plus one standard deviation) and the 97.5 percentile volume (i.e. mean plus two standard deviations).

The authors found that the use of average volume to capacity ratio tends to understate level of service (LOS) at busy intersections and concluded that for intersections operating at LOS D, a 10 % increase in traffic volumes would cause deterioration to LOS E or LOS F, about 15% of the time.

The authors also concluded that “it is desirable to base intersection service level computations on several days’ peak hour volumes”. However they did not make any recommendations regarding how many days or how this could be computed.

An earlier study by Kamarajugadda and Park (2003) proposed an analytical method to compute the impact of day-to-day variability of peak hour volume on variance of delay. Two separate estimation methods were developed – one for under-saturated intersections and the other for over-saturated intersections. The under-saturation model uses the concept of expectation functions to analytically relate the mean, variance, and distribution of peak hour volume to the mean and variance of delay. In this model, the HCM delay expression (Equation 1) is approximated using the Taylor Series expansion technique. This approach is not applicable to over-saturated conditions as the HCM delay expression is discontinuous at degree of saturation equal to 1.0. Consequently, the method of expectation functions cannot be used for degree of saturation ≥ 1.0 . Kamarajugadda and Park define an under-saturated critical lane group as one for which the 99.99th percentile degree of saturation is less than 1.0. The over-saturated model numerically integrates the expectation function over the range of degree of saturation from 0 to 3.

Kamarajugadda and Park validated their proposed models using Monte Carlo Simulation (MCS) for a hypothetical intersection. The validation consisted of comparing the mean and variance of delay estimated by the proposed models with the mean and variance of delay resulting from the MCS. The validation was conducted assuming that delay follows a Normal distribution.

Though Kamarajugadda and Park do not provide any statistical measures of validity, they conclude that their proposed models well represent the MCS results. However, they also note that the accuracy of their proposed models for estimating the mean delay decreases when degree of saturation exceeds 0.6. Furthermore, they also conclude that the variance of delay is substantially influenced by the distribution of peak hour volume that is assumed but they do not provide any recommendation on which distribution is most appropriate.

Given the available literature, it appears that the following questions remain to be addressed:

1. What is the magnitude and distribution of day-to-day variability in peak hour intersection approach volumes?
2. To what extent does the variability in peak hour approach volumes impact intersection delay?
3. How does the variability in day-to-day peak-hour delay influence the number of peak hour traffic volume counts required to estimate intersection performance with a given level of confidence?

The next section describes the empirical data used to address these questions.

EMPIRICAL VARIABILITY

DATA SET

Waterloo and Kitchener are adjacent cities located in south western Ontario, Canada approximately 120 km west of Toronto. The combined population of these two cities is 300,000. The regional government, which is responsible for traffic signal operations within these two cities, operates 16 continuous volume counting loop detector stations located mid-block on major arterial roadways.

Vehicle counts are obtained for each lane in both directions and aggregated at 15-minute intervals. Data from these vehicle count stations were obtained for the 2005 calendar year.

It is assumed that the volume counts from these stations can be interpreted as the approach volumes at the signalized intersections immediately downstream of the detector stations. This assumption implies that:

1. Any oversaturated conditions that may occur at the downstream signalized intersections do not cause queues to spill over the vehicle count stations for any significant portion of the 15 minute interval.
2. There are no significant mid-block flows (entering or leaving) between the vehicle count station and the downstream signalized intersection.

Local knowledge of the intersections in the vicinity of the volume counting stations suggests that this assumption is reasonable.

The individual lane data were aggregated to provide vehicle counts by direction (resulting in 26 directional volume count stations) and were filtered to remove data associated with weekends (i.e. Saturdays and Sunday) and all local and national holidays. This resulted in a maximum of 20,736 fifteen minute volume observations for each volume count station. However, as a result of hardware and communication system failures, some stations provided only a portion of these data. Stations with less than 70% data availability (i.e. fewer than 14,515 fifteen minute volume counts) were eliminated from the analysis. The remaining 13 stations exhibited an average annual daily traffic volume ranging from a low of 8,000 vehicles to 30,000 vehicles per non-holiday week day.

Typically, traffic engineers consider the PM peak period to be the highest demand period of the day and therefore, only data from 3:45 PM to 6:30 PM were considered for further analysis.

For each of the 13 stations, for each day, the volume count data were examined to determine:

1. Time of the start of the peak hour;
2. PM peak hour volume; and
3. Peak hour factor.

As a second quality check, the day-to-day variation in peak hour volume was examined for each of the 13 stations. Three stations exhibited erratic variation indicative of abnormal influences, such as lane closures due to construction during a portion of the year long data collection period. These stations were removed from the data set to avoid biasing the analysis.

VARIATION IN PEAK-HOUR VOLUME

Table 1 provides descriptive statistics for the peak hour volumes determined for the remaining 10 volume count stations. The mean peak hour volume varies significantly from one station to the next (i.e. ranging from 594 vph to 1375 vph), however, this variation is attributable to different traffic patterns on different roads, and is not of interest with respect to random day-to-day variations.

What is of interest, however, is the day-to-day variation in the peak hour volume that occurs at each site. This variation can be quantified by the coefficient of variation (*COV*) which is computed as the ratio of the standard deviation over the mean. The *COV* varies from a minimum of 5.4% to a maximum of 13.1% and on average is equal to 8.7%.

Sullivan *et al.*, conducted a similar analysis using data from the City of Milwaukee and found that the *COV* varied between approximately 5% and 16%. They suggested that the *COV* decreases with increasing mean volume but they did not fit a statistical model to confirm this. Figure 1 presents the *COV* of peak hour volume as a function of peak hour volume for both the Waterloo data and the City of Milwaukee data from Sullivan et al. (2006).

The Waterloo data and the City of Milwaukee data appear to be quite similar (means and variances of COV are the same at the 95% confidence level as determined using the t-test and F-test, respectively), though the Milwaukee data encompasses a larger range of peak hour volumes. As suggested by Sullivan, the data appear to exhibit a weak trend of decreasing COV with increasing peak hour volume.

Least squares linear regression was used to fit a linear model to the combined Waterloo and Milwaukee data resulting in.

$$COV = 0.129 - 0.036V \quad (6)$$

Where:

COV = coefficient of variation of the peak hour approach volume

V = mean peak hour approach volume

Though the regression intercept and coefficient are statistically significant at the 95% level (Student-t statistic = 7.7, P-value = 0.00000; Slope: Student-t statistic = -2.55, P-value = 0.016; Regression: F-value = 6.52, P-value = 0.0156), the regression explains only a small portion of the variance within the data (adjusted $R^2 = 0.15$) and therefore must be viewed with scepticism. It is possible that other non-linear model forms may marginally improve the model fit; however, it is clear from Figure 1 that no model that relies solely on mean peak hour directional volume will be able to explain a significant portion of the variance in the data. Given the relatively low explanatory power of the regression model and the weak association of COV with mean peak hour volume, the remainder of the analysis in this paper uses a constant value for COV equal to the mean of 0.087.

The COV can be used to characterise the variability within the approach volume distribution, however we also are interested to determine the shape of the distribution. This was accomplished by

normalizing each peak hour volume observation by dividing it by the mean peak hour volume for that volume count station. Consequently, it was possible to create distributions of normalized peak hour volumes and to compare these distributions for each of the 10 volume count stations (Figure 2).

The Kolmogorov-Smirnov (K-S) test was used to determine if each distribution could be adequately described by the Normal, Gamma, and Log-Normal distribution at the 99% level of confidence. It was found that the 10 distributions of day-to-day normalized peak hour volume are best described by the Normal distribution with a mean of 1.0 and a standard deviation of 0.087.

CORRELATION OF PEAK HOUR VOLUMES

In the previous section, it was determined that the day-to-day variation in weekday peak hour volumes can be modelled by a normal distribution with a coefficient of variation of 0.087. However, there remains the question of whether or not the peak hour traffic demands on each intersection approach are statistically correlated. The volume count data represented mid-block flows from various locations throughout Waterloo region. Consequently, it was not possible to directly determine the correlation between traffic volumes on different approaches to the same intersection. Nevertheless, it was possible to test the extent to which peak hour traffic volumes at different mid-block locations are correlated. A high correlation could be interpreted to mean that when peak hour traffic demands are higher than average they tend to be higher than average at all locations including all approaches to an intersection.

The correlation coefficient ρ was computed between the peak hour volumes for each pair of stations (Table 2). The value of ρ ranged from 0.003 to 0.55 with an average of 0.3 indicating that in general the level of correlation is relatively weak. This suggests that when peak hour traffic volumes on one approach are much lower (or higher) than average there is not a high likelihood that peak hour volumes on the other approaches are also lower (or higher) than average.

In an effort to explore whether or not these results might be consistent with the correlation of peak hour volumes on different approaches to the same intersection, an additional set of 15-minute aggregated traffic counts was obtained from the City of Toronto which also operates a set of permanent count stations distributed across the arterial network. Four of these permanent count stations are located on the approaches to a four legged signalized intersection (Brimley Road and Pitfield Road). The peak hour volumes for all non-holiday weekdays in 2006 were extracted and the correlation coefficient between volumes for each pair of approaches was computed. The correlations ranged from 0.20 to 0.57 with an average of 0.33 which corresponds quite closely to the average correlation computed from the Waterloo data.

It is recognised that this comparison is based on only a single intersection and is therefore not conclusive. Nevertheless, it appears imprudent to assume that the correlation between peak hour volumes on different approaches is zero, given that doing so would tend to over-estimate the variability of delay.

Further work is required to confirm that a similar range of correlation exists between peak hour volumes on different approaches to a broader range of signalized intersections.

VARIATION IN TIME OF PM PEAK HOUR

The variation in the time at which the PM peak hour volume occurred was also examined. On average over all 10 stations, the peak hour volume occurred between 4:30 and 5:30 PM with a standard deviation of approximately 20 minutes.

From a signalized intersection analysis and design perspective the time of the peak hour is likely not of high importance. Whether the peak hour begins at 4:30 PM or at 5 PM on a particular day is less important than the performance of the intersection during the peak hour. However, these results do

suggest that when the collection of field data reflecting peak volume conditions is necessary, the time of occurrence of the peak hour cannot be assumed to be fixed from one day to the next.

VARIABILITY OF INTERSECTION DELAY AND LOS

The objective of this section is to explore the impact that the day-to-day variability of peak hour volumes has on the operating characteristics of a typical 4-leg intersection operating under a fixed time traffic signal control strategy. Intersection delay is difficult to measure accurately in the field and it is cost prohibitive to do so for a number of intersections over a large number of days. Consequently, in this study, (and as is typically done in practice) intersection delay was estimated using the Highway Capacity Manual methodology. The following sections describe the hypothetical intersection was developed for this study, followed by the description of Monte Carlo simulation used to evaluate the intersection performance and finally the results are presented and discussed.

HYPOTHETICAL INTERSECTION

A hypothetical 4-leg intersection was assumed. Each approach consisted of an exclusive left turn lane, an exclusive through lane, and a shared through and right turn lane. All lane widths, grade, curb radii, etc. were considered to be ideal with no on-street parking, no transit vehicles, and adequate storage and discharge space. The base saturation flow rate was assumed to be 1900 passenger cars per hour per lane (pcphpl). The intersection was controlled by a two-phase signal timing plan with a cycle length of 80s; 38s effective green for phase 1; 34s effective green for phase 2; and 4 seconds of inter green between each phase. Right-turn on red was not permitted.

Eleven traffic demand scenarios were developed encompassing intersection volume to capacity (v/c) ratios ranging from 0.6 to 1.10. For each scenario, the turning movement proportions remained constant (1% left turn, 79% through, and 20% right turn) but the total approach demands varied (Table 3). For all cases, the traffic stream was assumed to consist of only passenger cars.

MONTE CARLO SIMULATION

The performance of the hypothetical intersection, in terms of average vehicle delay, was evaluated using the methodology defined by the HCM. It should be noted that any other similar analytical method (e.g. Canadian Capacity Guide) for estimating average vehicle delay could have been used instead of the HCM method and similar results would have been obtained (although this has not been confirmed). The following parameter values required within the HCM methodology were assumed:

- Evaluation time period = 0.25 hours
- $PHF = 0.923$
- Area type = 1 (Central Business District, CBD)
- Arrival type = 4

For each of the 11 demand scenarios, 1000 Monte Carlo trials were evaluated. For each Monte Carlo trial, peak hour approach volumes were generated randomly using a Normal distribution with a $COV = 0.087$ and the mean peak hour volume from Table 3. The volumes on each approach were generated to be correlated with $\rho = 0.3$.

For all simulations, the signal timing plan, saturation flow rate, PHF , turning movement proportions and all other inputs except the approach volumes remained unchanged. For each Monte Carlo trial, the HCM methodology was used to estimate the average delay during the peak hour.

All simulation runs were conducted using Crystal BallTM version 7.2.2 combined with the HCM methodology implemented within Excel.

RESULTS

Figure 3 illustrates the cumulative distribution of average intersection delay associated with each of the eleven traffic demand scenarios. Figure 4 illustrates the associated standard deviation of peak

hour average delay as a function of the mean delay for each of the 11 demand scenarios. The standard deviation increases dramatically as the mean delay increases. The effect of this is illustrated in Figure 5 which depicts the mean, 95% and 99% confidence limits (i.e. 2.5, 97.5 percentile and 0.5, 99.5 percentile of the Monte Carlo simulation results) associated with the intersection delay. Several observations can be made on the basis of these results.

First, as expected, the variation in the intersection delay increases dramatically as the intersection v/c ratio increases. For example, consider the 95% confidence limits of the intersection delay when $v/c=0.6$. It is expected that 95% of the time, the peak hour intersection delay will be between 14.8 and 17.1 seconds/vehicle (LOS B). However, for $v/c=0.9$, the 95% confidence limit is from 21.6 to 51.9 seconds/vehicle (LOS C and D).

Second, the distribution of intersection delay appears to be generally log-Normally distributed. This was confirmed by the K-S test which showed that 9 of the 11 scenarios could be described by a log-Normal distribution at the 99% level of confidence.

Figure 6 illustrates the impact that the non-linear relationship between volume and delay has on estimating the mean intersection delay. The x-axis is the volume to capacity ratio as specified in Table 3. The left-hand y-axis is the estimated intersection delay and the right-hand y-axis is the estimation error computed as:

$$Estimation\ Error = 100\% \times (d - d')/d \quad (7)$$

where:

d = mean intersection delay computed using the average approach volumes

d' = mean intersection delay computed as the average of the delays obtained from the 1000 Monte Carlo simulation trials.

When the estimation error is equal to zero, both methods provide the same estimate of average intersection delay. The estimation error is small (but positive) for low v/c ratios and increases as v/c approaches 1.0 to a maximum value of approximately 20% and then begins to decrease as v/c continues to increase. For all the v/c scenarios examined, the estimation error is positive indicating that computing the intersection delay on the basis of the average volumes, and ignoring the variability of these volumes, under-estimates the true average intersection delay by as much as 20%.

The results in Figure 6 can be explained using Figure 7 which depicts the typical relationship of intersection delay as a function of volume to capacity ratio (degree of saturation). If we assume the capacity of the intersection is fixed (i.e. signal timings are not changed) then the x-axis can be thought of as volume.

Distribution A represents the day-to-day distribution of volume for a relatively low value of X . Distribution a represents the corresponding delay distribution. The mean of distribution a is very similar to the delay obtained for the mean of distribution A because the relationship between delay and X is nearly linear over the range of distribution A . Consequently, observations over this range correspond to a ratio close to 1.0 in Figure 6.

Distribution B represents the day-to-day distribution of volume for X approximately equal to 1. Distribution b represents the corresponding delay distribution. The mean of distribution b is clearly larger than the delay obtained for the mean of distribution B . This result is obtained because the increase in delay for volumes greater than the mean is much larger than the decrease in delay

associated with volumes smaller than the mean (i.e. rate of change in curvature (i.e. the second derivative) of the delay equation 2 or 6 is maximum at $X = 1$).

Distribution C represents the day-to-day distribution of volume for conditions of $X > 1$. Distribution c represents the corresponding delay distribution. The mean of distribution c is very similar to the delay obtained for the mean of distribution C because the relationship between delay and volume is again nearly linear.

Therefore, in general the largest estimation error occurs for conditions in which the intersection degree of saturation is near to 1 the conditions which are most likely to require intersection improvements.

The preceding discussion has focussed on the estimating the distribution of intersection delay given that the true mean peak hour volume or the distribution of peak hour approach volumes is known. Of course in practice, the true mean and the distribution of peak hour volume is rarely known. In practice, typically peak hour turning movement traffic counts are obtained from a single day (or in rare cases on a few days) and these counts are used within the HCM method to estimate the intersection delay. The traffic counts obtained on a single day represent a single observation from the distribution of peak hour volumes. Given the variability in the intersection performance, a single observation is rarely adequate to reliably estimate the average intersection delay. The obvious question then is how many days of observations are required.

The number of days of observations required can be estimated through the use of a two-stage sampling process. In the first stage of the sampling process, a sample of n_1 days is randomly selected. The mean and standard deviation of the average peak hour intersection delay for the sample can be computed using Equations 8 and 9 respectively.

$$d'' = (1/n_1) \sum_{i=1}^{n_1} d_i \quad (8)$$

Where:

n_1 = number of days of observations of peak hour volume in sample

d'' = mean intersection delay for sample

d_i = average intersection delay for peak hour on day i

$$s = \text{SQRT}((1/(n_1-1)) \sum_{i=1}^{n_1} (d_i - d_a)^2) \quad (9)$$

Where:

s = sample standard deviation of the average peak hour intersection delay

If it is assumed that the distribution of the sample means (i.e. d_a) follows a Normal (or nearly Normal) distribution, then the confidence limits on the estimate of the true (population) intersection delay can be expressed as

$$d_a - t_{n_2-1, \alpha} (s / \text{SQRT}(n_2)) \leq D \leq d_a + t_{n_2-1, \alpha} (s / \text{SQRT}(n_2)) \quad (10)$$

Where:

n_2 = number of days of observations of peak hour volume in sample

D = true (unknown) mean peak hour intersection delay

t = Student-t statistic

Equation 13 can be re-arranged to express the number of observations required to achieve a desired estimation error with a specified level of confidence.

$$n_2 = ((t_{n_2-1, \alpha} s) / \varepsilon)^2 \quad (11)$$

Where:

n_2 = required number of days of observations of peak hour volume

$t_{n_2-1, \alpha}$ = student t distribution value for n_2-1 degrees of freedom and a probability of α

S = sample standard deviation of intersection delay computed from the initial sample

ε = desired maximum error in the estimation of the true mean intersection delay

We establish an estimate of the required number of days of traffic counts by applying the two stage sampling technique to the average intersection delays generated through the Monte Carlo simulations described previously in this paper. Average intersection delays are available for 1000 trials for each of the 11 v/c scenarios. Each trial represents the performance of the hypothetical intersection during a random PM peak hour. For this study, n_1 was held constant at 3.

Values of ε between 10% and 50% of the true average intersection delay (D) were examined. The two stage sampling process was conducted 1000 times for each level of ε considered and the average value of n_2 recorded.

The results of this analysis are depicted in Figure 8 which shows the number of days of observations required to achieve a selected maximum estimation error. For example, if a maximum estimation error of 30% of the true mean intersection delay is acceptable, then the number of days for which peak hour volumes is required to be observed ranges from 1 at $v/c \leq 0.7$ to 7 at $v/c \approx 1.0$. Obviously, if a more accurate estimate of the mean intersection delay is required (e.g. 20% error), then more observations of peak hour volumes must also be obtained. The choice of the acceptable level of error represents a trade-off between accuracy or reliability of the estimated intersection performance and the cost of acquiring turning movement counts. Current practice typically is to conduct volume counts on a single day; implying reliability of the estimated intersection performance may be very poor (i.e. error may be greater than 50% of the true average delay for $v/c > 0.85$). Given that decisions associated with intersection improvements (and possibly developer fees) may be made on these intersection performance estimates, greater reliability of the estimates may be necessary.

The benefit of taking multiple traffic counts can be illustrated by examining the probability that the intersection level of service (LOS) estimated from the HCM method will be better than the true average intersection LOS. Figure 9 illustrates the probability that the estimated LOS is better than the true LOS for two different cases. In the first case, a single set of peak hour traffic counts are obtained and used to estimate the intersection LOS. In the second case, the number of peak hour traffic counts obtained varies as a function of the v/c ratio (as per Figure 8). For both cases, 1000 repetitions were conducted. For each repetition, the sample was randomly selected from the peak hour approach volumes generated for the hypothetical intersection in the Monte Carlo simulations. The estimated intersection LOS was determined from the average of the intersection delays determined for each observation in the sample.

As expected, Figure 9 demonstrates that for all values of v/c taking multiple samples provides a more reliable estimate of the true intersection LOS than using traffic counts from a single day.

For $v/c \leq 0.7$, the use of traffic counts from a single day is sufficient to estimate the intersection LOS with virtual certainty (i.e. probability that the estimated LOS will be equal to the true LOS is almost 100%). As the v/c ratio increases, the level of confidence that the LOS estimated from traffic counts observed on a single day is in fact equal to the true intersection average LOS, declines significantly. At a v/c ratio of between 0.95 and 1.1, there is approximately a 50% chance that the estimated LOS is actually correct.

When traffic counts from multiple days are taken and used to estimate the intersection LOS, the likelihood that the estimate is correct increases. The irregular relationship between LOS estimation accuracy and v/c ratio occurs as a result of several factors including: (a) LOS is defined by discrete values of delay and different levels of service are associated with different ranges of delay; (b) the distribution of intersection delay is not symmetrical but is skewed to the right (as illustrated in Figure 3); and (c) the number of traffic counts used to estimate the intersection LOS for the “multiple observations” case varies with degree of saturation (as indicated in Figure 9).

CONCLUSIONS AND RECOMMENDATIONS

The day-to-day variability of peak hour approach volumes is not considered within signal evaluation and design methodologies. Rather, the typical practice is to determine intersection performance, in terms of average vehicle delay, on the basis of peak hour volumes observed on a single day.

In this study, we have determined on the basis of empirical data that:

1. The day-to-day variation of weekday peak hour volumes can be represented by a Normal distribution with a coefficient of variation of 0.087. These findings are consistent with the findings of Sullivan et al (2006).
2. The coefficient of variation of peak hour volumes appears to decrease as the mean peak hour volume increases. A linear model was found to be statistically significant, however, it explains only a small fraction of the variance in the observed data (adjusted $R^2 = 0.15$).
3. The variation of peak hour approach volumes appear to exhibit a relatively weak statistical correlation (mean $\rho = 0.3$).
4. For a theoretical intersection with a specified set of operating conditions the estimation of average intersection delay on the basis of average peak hour volumes under-estimated the true delay averaged across 1000 trials of volume variations by as much as 20%. Furthermore, the greatest underestimation error occurs for intersections operating in the range of $X \approx 1$. Depending on the signal timing g/C ratio, this can be associated with an intersection operating at LOS D or even C.
5. The number of days of observations of peak hour volumes required to estimate intersection average performance during the peak hour of a typical weekday was established as a function of the desired level of accuracy.

On the basis of these observations and conclusions, the following recommendations are made:

1. Additional field data should be obtained from another region to confirm the findings of this study.
2. The impact on intersection performance of day-to-day variability of other factors such as the PHF and turning movement proportions should be examined.

3. The presence of significant auto-correlation in the intersection peak hour traffic volumes could impact the number of observations required to estimate intersection performance within a specified tolerance. Consequently, an analysis should be conducted to quantify the extent of auto-correlation and the impact that this has.
4. Criteria should be established to incorporate the day-to-day variability of volume within existing signalized intersection evaluation and analysis methodologies.

ACKNOWLEDGEMENTS

The authors wish to acknowledge Mark Liddell from the Regional Municipality of Waterloo and Blair Lagden for providing the system detector data used in this study. The authors also wish to acknowledge the following agencies/organization that provided financial support for this project:

- Transport Canada, Urban Transportation Showcase Program
- Regional Municipality of Waterloo, and
- Natural Science and Engineering Research Council of Canada.

REFERENCES

- Allsop, R. E. (1972). "Delay at a Fixed Time Traffic Signal – I: Theoretical Analysis". *Transportation Science*, Vol. 6, 1972, pp. 260-285.
- Colyar, J.D. and N.M. Roupail (2003). "Measured Distributions of Control Delay on Signalized Arterials." *Transportation Research Record* 1852, TRB, National Research Council, Washington D. C., pp. 1-9.

Cronje, W.B. (1983). "Derivation of Equations for Queue Length, Stops, and Delay for Fixed-Time Traffic Signals." *Transportation Research Record* 905, TRB, Washington D. C., 1983.

Engelbrecht, R. J., D. B. Fambro, N. M. Rouphail, and A. A. Barkawi. (1997). "Validation of Generalized Delay Model for Oversaturated Conditions." *Transportation Research Record* 1572, TRB, National Research Council, Washington, D.C., pp. 122-130.

Fu, L. and B. Hellings (2000). "Delay variability at signalized intersections", *Transportation Research Record* 1710, TRB, National Research Council, Washington, D.C., pp. 215-221.

Hurdle, V. F. (1984). "Signalized Intersection Delay Models—A Primer for the Uninitiated." *Transportation Research Record* 971, TRB, National Research Council, Washington, D.C., pp. 96-105.

Kamarajugadda, A. and B. Park (2003). "Stochastic Traffic Signal Timing Optimization.", *Final report of ITS Center project: Signal timing algorithm For the Center for ITS Implementation Research*, Anil Kamarajugadda and Byungkyu (Brian) Park., A U.S. DOT University Transportation Center, Department of Civil Engineering, Center for Transportation Studies, University of Virginia.

Newell, G. F. (1982). "Applications of Queueing Theory.", University Press, Cambridge, England.

Olszewski, P. (1993) "Modeling of Queue Probability Distribution at Traffic Signals. In *Transportation and Traffic Theory*, ed. M. Koshi, Elsevier, New York.

Olszewski, P. (1994). "Modeling Probability Distribution of Delay at Signalized Intersections." *Journal of Advanced Transportation*, Vol. 28, No. 3, pp. 253-274.

"Highway Capacity Manual" (2000). Transportation Research Board of the National Academies Washington, D.C.

Sullivan, D., Levinson, H.S., Bryson, R.W. (2006). "Effects of Urban Traffic Volume Variations on Service Levels." Paper # 06-2381, *proc. 85th annual Transportation Research Board meeting*, Washington, D.C., January 2006

Teply, S., and G. D. Evans (1989). "Evaluation of the Quality of Signal Progression by Delay Distributions." *Transportation Research Record* 1225, TRB, National Research Council, Washington, D.C., pp. 1-7.

Teply, S., D. I. Allingham, D. B. Richardson and B. W. Stephenson (1995). "Canadian Capacity Guide for Signalized Intersections, Second Edition." (S. Teply, ed.), Institute of Transportation Engineering, District 7, Canada, 1995 (ISBN 0-935403-09-4).

Webster, F. V. (1958). "Traffic Signal Settings." *Technical Paper No. 39*, Road Research Laboratory, London, U.K.

LIST OF TABLES

Table 1: Peak hour volume descriptive statistics

Table 2: Correlation Matrix for approach volumes

Table 3: Evaluation scenarios

Table 1: Peak hour volume descriptive statistics

Volume Count Detector Station	Mean (vph)	Standard Deviation (vph)	Coefficient of Variation	Observation	Maximum (vph)	Minimum (vph)	Number of Lanes
62 (North Bound)	1287	69.6	0.054	209	1448	1042	3
182 (West Bound)	1375	97.6	0.071	213	1671	811	2
184 (West Bound)	658	61.5	0.094	213	1047	454	2
184 (East Bound)	594	54.9	0.093	213	788	277	2
290 (West Bound)	1282	111.5	0.087	213	1750	996	2
312 (North Bound)	971	62.6	0.065	214	1160	746	2
313 (North Bound)	822	52.7	0.064	214	987	640	2
313 (South Bound)	855	112.3	0.131	214	1033	564	2
484 (North Bound)	720	69.4	0.096	204	1134	558	2
484 (South Bound)	961	106.6	0.111	171	1193	490	2
Overall Average	952	79.9	0.084	208	1750	277	

Table 2: Correlation Matrix for approach volumes

Station	62 (North Bound)	182 (West Bound)	184 (West Bound)	184 (East Bound)	290 (West Bound)	312 (North Bound)	313 (North Bound)	313 (South Bound)	484 (North Bound)	484 (South Bound)
62 (North Bound)	1.00									
182 (West Bound)	0.37	1.00								
184 (West Bound)	0.41	0.05	1.00							
184 (East Bound)	0.39	0.09	0.47	1.00						
290 (West Bound)	0.38	0.22	0.31	0.00	1.00					
312 (North Bound)	0.55	0.35	0.38	0.32	0.44	1.00				
313 (North Bound)	0.43	0.30	0.36	0.52	0.02	0.39	1.00			
313 (South Bound)	0.35	0.54	0.36	0.37	0.31	0.45	0.53	1.00		
484 (North Bound)	0.44	0.12	0.26	0.21	0.32	0.32	0.21	0.12	1.00	
484 (South Bound)	0.33	0.24	0.14	0.25	0.00	0.17	0.40	0.07	0.09	1.00

LIST OF FIGURES:

Figure 1: Coefficient of variation of peak hour volume as a function of peak hour volume

Figure 2: Distributions of normalized peak hour volume

Figure 3: Distribution of intersection delay as a function of v/c ratio

Figure 4: Standard deviation of average delay as a function of peak hour mean delay

Figure 5: Mean and confidence limits of intersection peak hour delay

Figure 6: Error in intersection delay estimation when using mean approach volumes instead of considering the variation of peak hour volume

Figure 7: Effect of non-linear delay relationship on estimates of mean delay

Figure 8: Number of days of peak hour volume observations required to achieve specified accuracy of average intersection delay

Figure 9: Effect of multiple samples on improving the likelihood that the estimated LOS will be the same as the true LOS

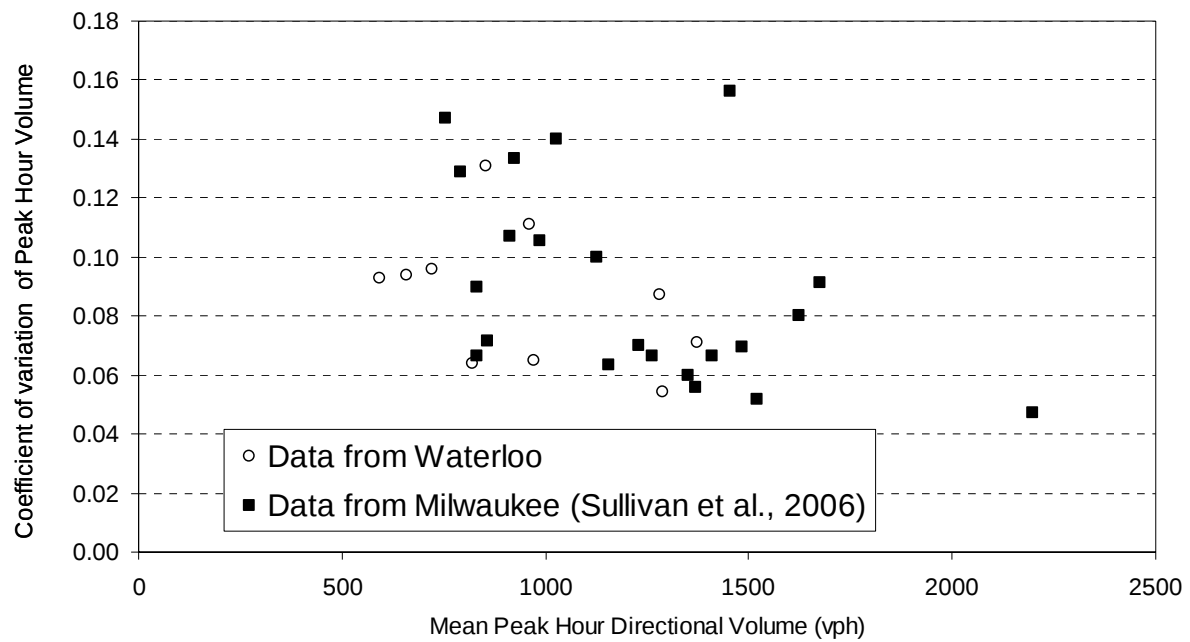


Figure 1: Coefficient of variation of peak hour volume as a function of peak hour volume

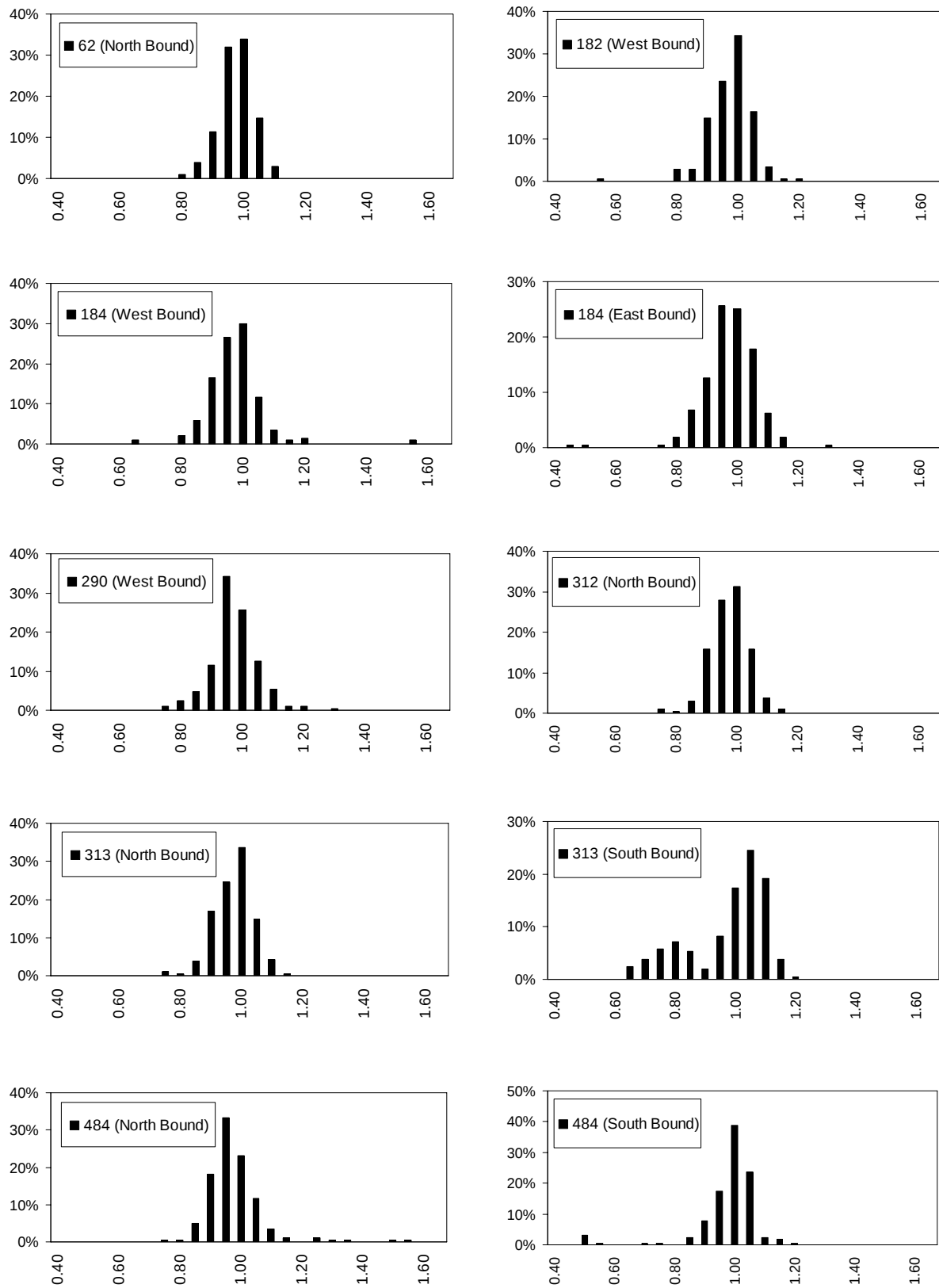


Figure 2: Distributions of normalized peak hour volume

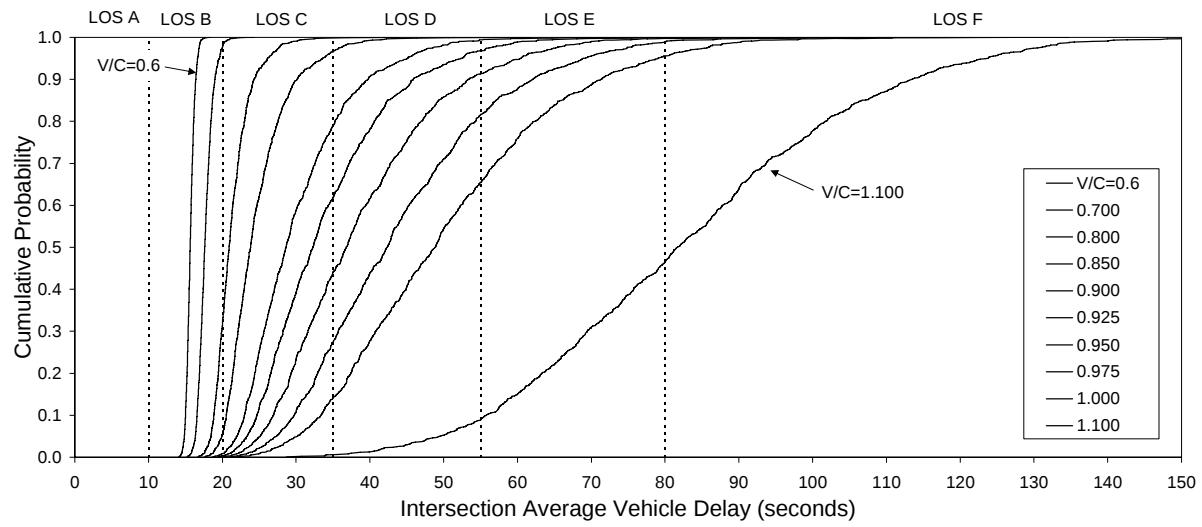


Figure 3: Distribution of intersection delay as a function of v/c ratio

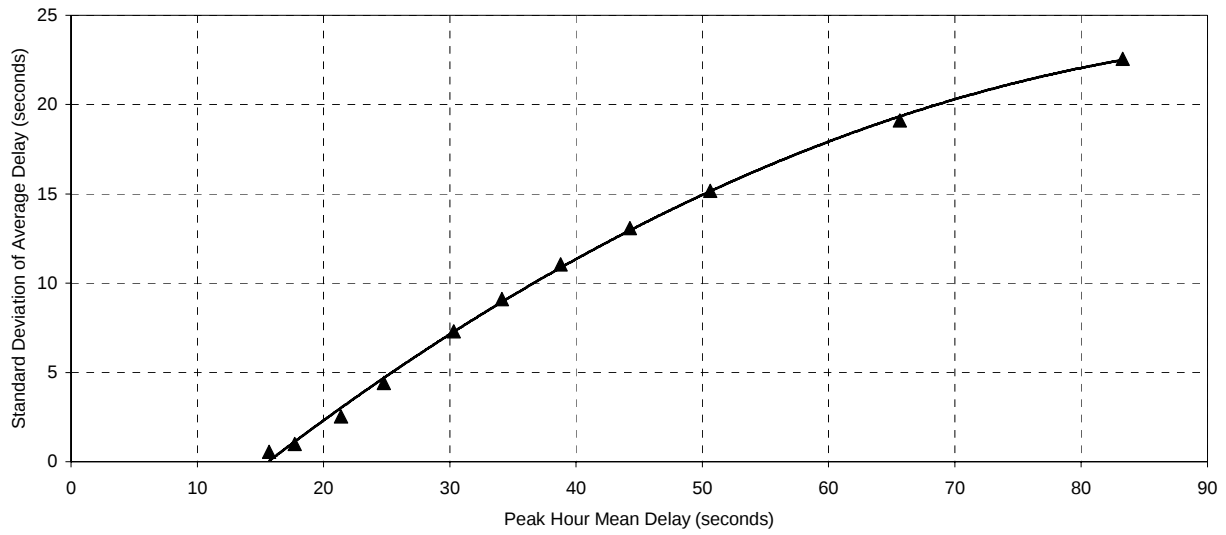


Figure 4: Standard deviation of average delay as a function of peak hour mean delay

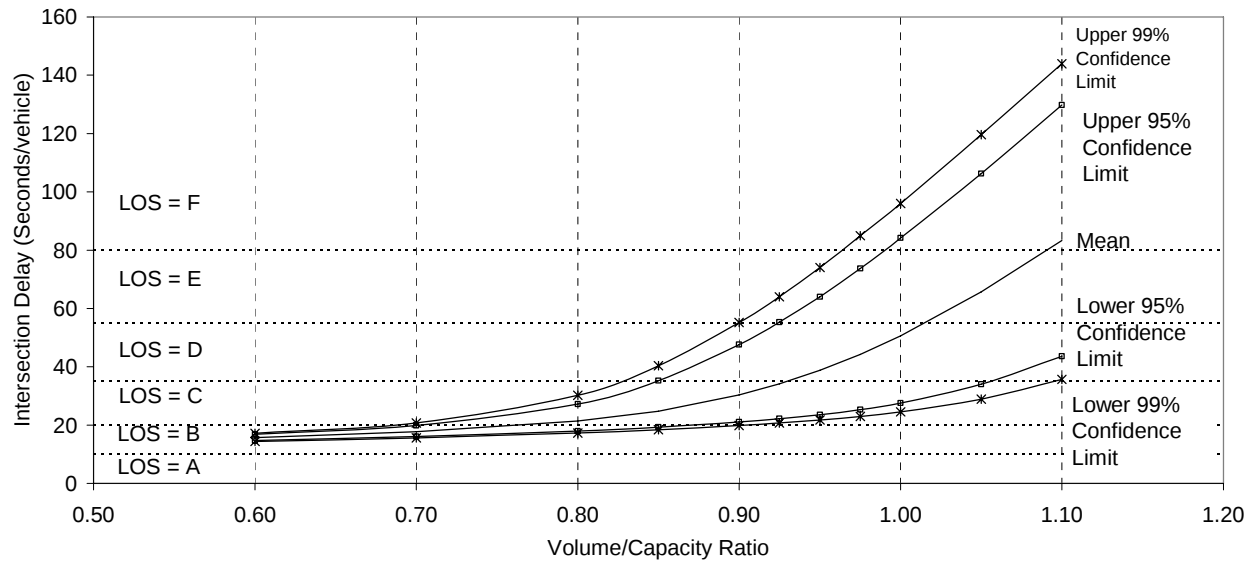


Figure 5: Mean and confidence limits of intersection peak hour delay

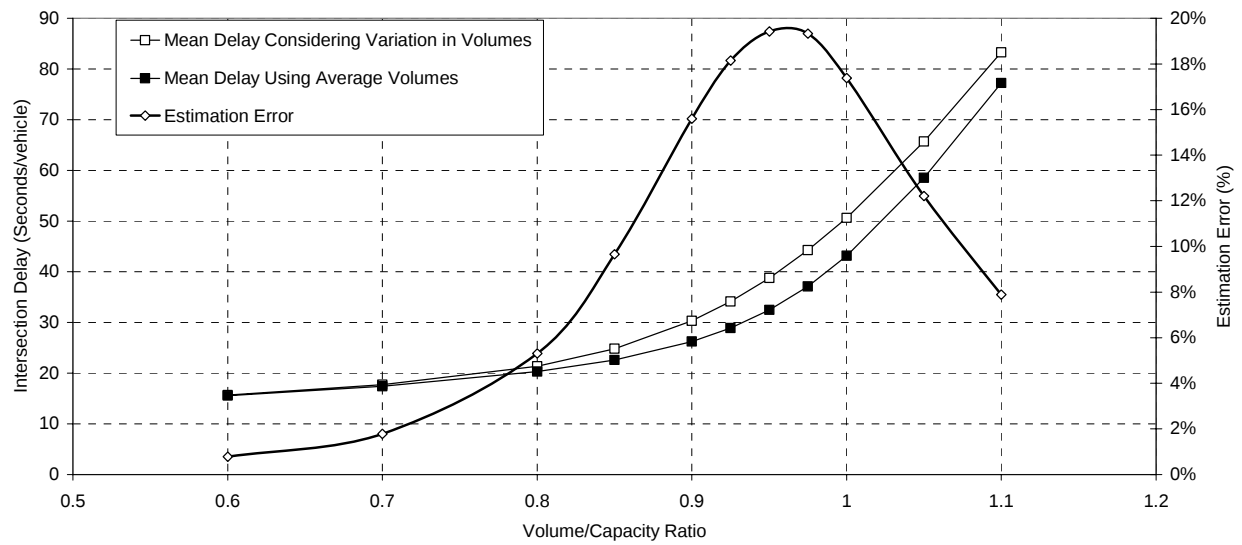


Figure 6: Error in intersection delay estimation when using mean approach volumes instead of considering the variation of peak hour volume

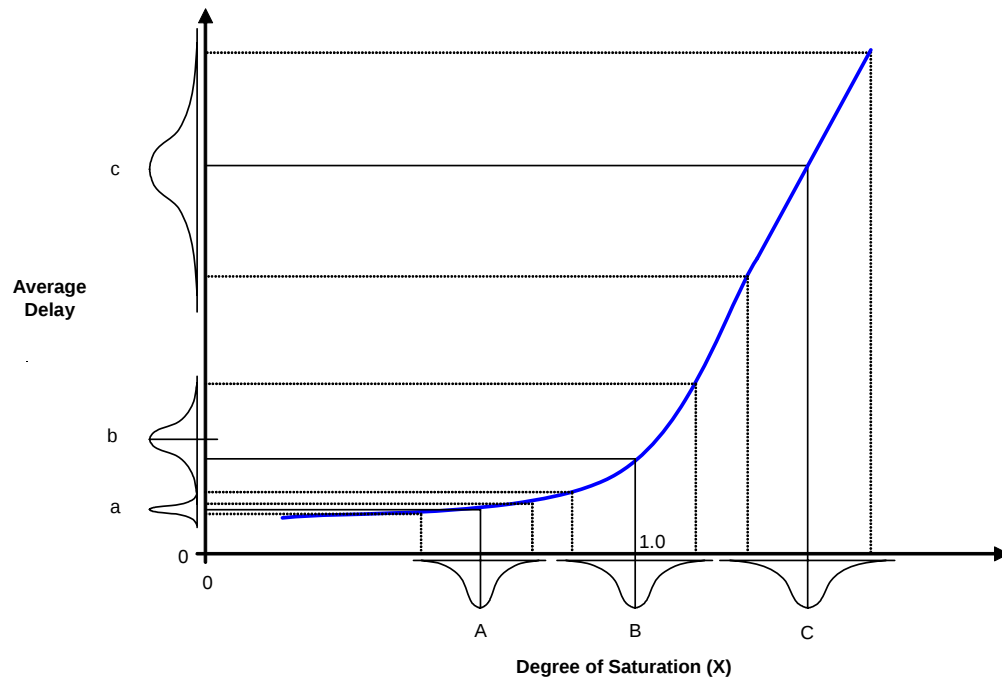


Figure 7: Effect of non-linear delay relationship on estimates of mean delay

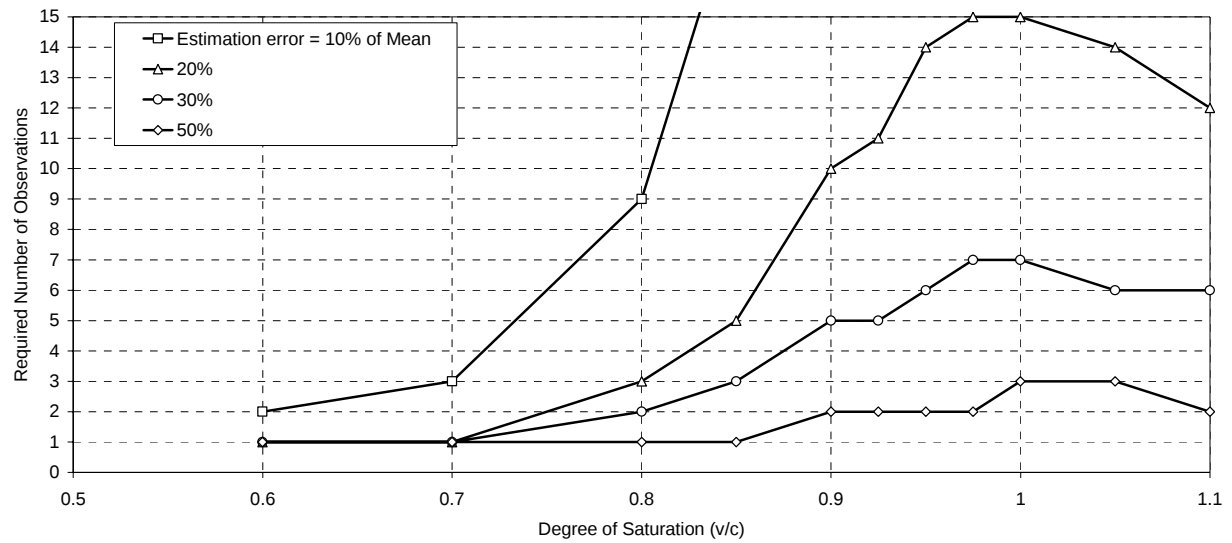


Figure 8: Number of days of peak hour volume observations required to achieve specified accuracy of average intersection delay

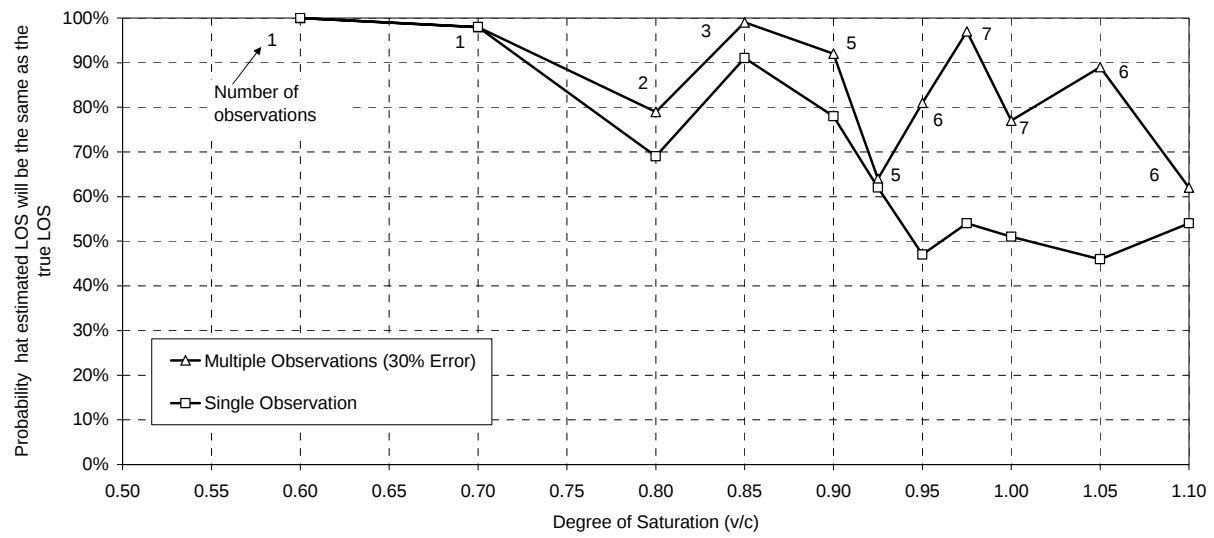


Figure 9: Effect of multiple samples on improving the likelihood that the estimated LOS will be the same as the true LOS